

Test 3A - MTH 2010  
Dr. Graham-Squire, Spring 2017

Name: Key

I pledge that I have neither given nor received any unauthorized assistance on this exam.

---

(signature)

## DIRECTIONS

1. Show all of your work and use correct notation, even on multiple choice questions! A correct answer with insufficient work or incorrect notation will lose points.
2. Clearly indicate your answer by putting a box around it.
3. Calculators, cell phones and computers are not allowed on this test.
4. Make sure you sign the pledge.
5. Number of questions = 14. Total Points = 70.

1. (5 points)

- (a) (1 point) Snail A moves 5 inches every 4 minutes. Snail B moves 4 inches every 3 minutes. Which snail moves faster? Explain your reasoning/show your work.

|   |     |   |    |    |
|---|-----|---|----|----|
| A | in  | 5 | 10 | 15 |
|   | min | 4 | 8  | 12 |

|   |     |   |   |    |    |
|---|-----|---|---|----|----|
| B | in  | 4 | 8 | 12 | 16 |
|   | min | 3 | 6 | 9  | 12 |

Snail B is faster, moves ~~1~~ ~~1~~ one more inch every 12 minutes.

- (b) (2 points) Adam has 13 eggs that he wants to use to make a cake that calls for 5 eggs. Explain the difference between the answers of "2, remainder 3" and " $2\frac{3}{5}$ " to figure out how many cakes Adam can make. In particular, do the 2, 3, and  $\frac{3}{5}$  all represent the same wholes, or are they different?

2 rem 3 means 2 cakes, 3 eggs left over

$2\frac{3}{5}$  means  $2\frac{3}{5}$  of a cake can be made.

So the 2 and  $\frac{3}{5}$  both have "cake" as the whole,

But the 3 "eggs" is a different whole

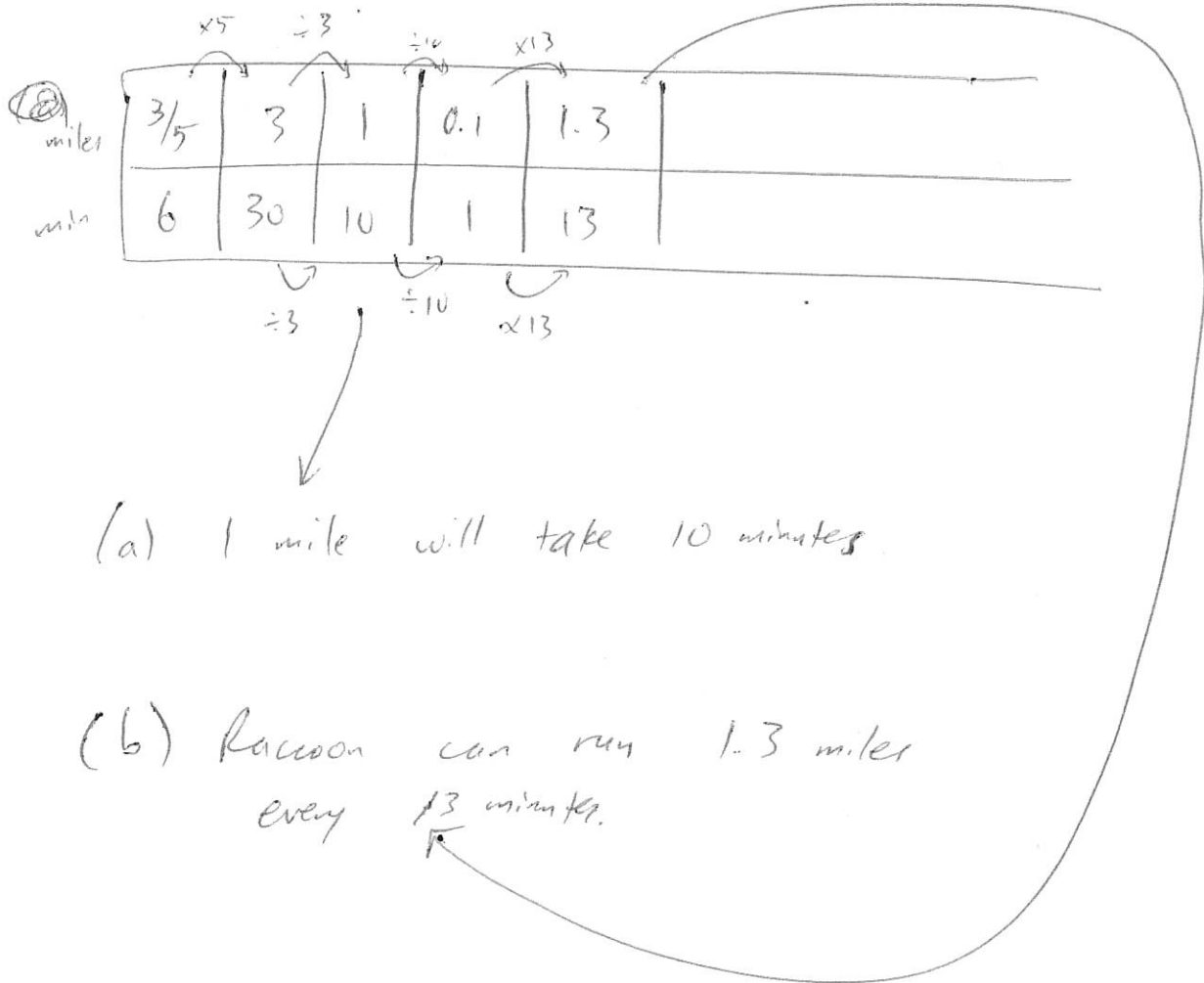
- (c) (2 points) It takes Theo 30 minutes to pick a peck of snide. It takes Judy 40 minutes to pick a peck of snide. Approximately how long will it take them to pick a peck of snide together? Note that you do NOT have to have the exact answer—as long as you explain your reasoning and your answer is within 3 minutes of the correct answer, you will get full points.

Between 15 and 20 minutes (half of each of their times), so 17.5 minutes is a good estimate.

2. (5 points) A raccoon runs  $\frac{3}{5}$  of a mile every 6 minutes.

(a) How long will it take the raccoon to run a mile? Show/explain your work.

(b) How far can the raccoon run in 13 minutes? Show/explain your work.



3. (5 points) Which of the numbers below is prime? Show/explain your work!

(A) 854  $\times$  divisible by 2

(B) 307

(C) 717  $\times$  divisible by 3 b/c  $7+1+7=15$

(D) 125  $\times$  divisible by 5

(E) 203  $\times$  divisible by 7

$$\begin{array}{r} 29 \\ 7 \overline{) 203} \\ \underline{14} \phantom{0} \\ 63 \\ \underline{-63} \\ 0 \end{array}$$

$$\begin{array}{r} 43 \\ 7 \overline{) 307} \\ \underline{-28} \phantom{0} \\ 27 \\ \underline{-21} \\ 6 \end{array}$$

$\times$

$$\begin{array}{r} 27 \\ 11 \overline{) 307} \\ \underline{-22} \phantom{0} \\ 87 \\ \underline{-77} \\ 10 \end{array}$$

$\times$

tip!  $\Rightarrow$  307 is prime

$$\begin{array}{r} 23 \\ 13 \overline{) 307} \\ \underline{26} \phantom{0} \\ 47 \\ \underline{-39} \\ 8 \end{array}$$

$\times$

$$\begin{array}{r} 18 \\ 17 \overline{) 307} \\ \underline{-17} \phantom{0} \\ 137 \\ \underline{-136} \\ 1 \end{array}$$

$\times$

$$\begin{array}{r} 16 \\ 19 \overline{) 307} \\ \underline{-19} \phantom{0} \\ 117 \\ \underline{-114} \\ 3 \end{array}$$

$\times$

4. (5 points) You are attempting to do the calculation  $1739 \div 215$ . Which of the following methods of approximation will get you closest to the correct answer? Note: you do not have to actually do each calculation (though that may help you decide), you just have to determine which one would give you the best approximation (by "best", I mean closest in value to the correct answer, not necessarily the fastest or easiest), and explain why.

- Round 1739 up to 1800, then do  $1800 \div 215$
- Round 215 down to 200, then do  $1739 \div 200$
- Round 1739 up to 1800 AND round 215 down to 200, then do  $1800 \div 200$

This will be closest b/c changing the denominator to 200 will make too many extra groups go into 1739

This is the worst b/c one rounds up and other is down

$$\begin{array}{r} 8.3 \\ 215 \overline{) 1800.0} \\ \underline{- 1720} \\ 80.0 \\ \underline{- 645} \\ 155 \end{array}$$

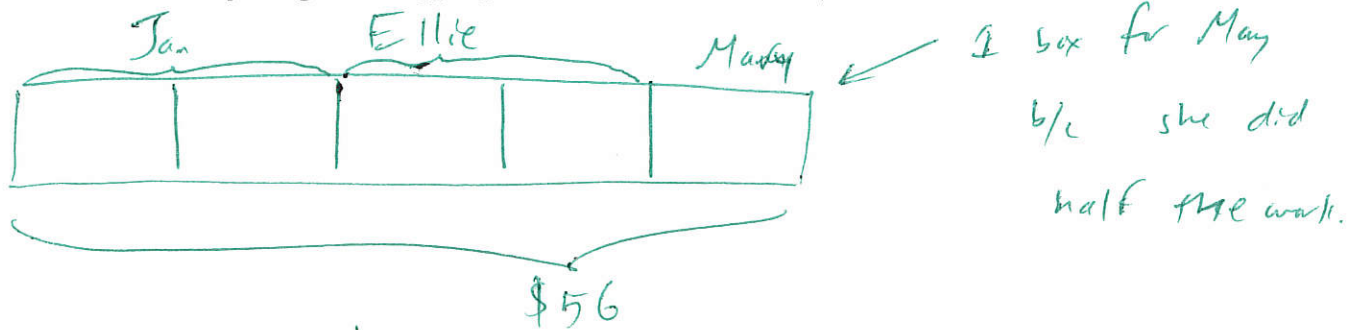
closest to

$$\begin{array}{r} 8.6 \\ 200 \overline{) 1739.0} \\ \underline{- 1600} \\ 1390 \end{array}$$

$$\begin{array}{r} 8.0 \\ 215 \overline{) 1739.0} \\ \underline{- 1720} \\ 190 \end{array}$$

$$1800 \div 200 = \underline{9}$$

5. (5 points) Mary, Jan and Ellie earned \$56 together raking leaves. Mary did not work as much as the others, though, so they all agree that Mary should get half as much as Jan and Ellie each get (Jan and Ellie worked the same amount). How should the \$56 be divided so they all get the appropriate amounts? Explain/show your work.



$\Rightarrow$  5 boxes = \$56

$\Rightarrow$  1 box =  $56 \div 5 = \$11.20$

So

|       |      |         |
|-------|------|---------|
| Jan   | gets | \$22.40 |
| Ellie | gets | \$22.40 |
| Mary  | gets | 11.20   |

2.5 for 21, 21, 14

4 for 22.5, 22.5, 11

6. (5 points) Consider the division problems  $3 \div 0$ ,  $3 \div 6$ ,  $0 \div 0$  and  $0 \div 3$ .

(a) Which of those divisions are defined and which one(s) are not defined?

(b) If a division is not defined, explain why. If more than one division is not defined, are they undefined for the same reason or for different reasons?

It may help to rewrite the division problems as multiplication by an unknown quantity.

(a)  $3 \div 6$  and  $0 \div 3$  are defined ✓

$3 \div 0$  and  $0 \div 0$  are not defined. ✓

(b)  $3 \div 0 = \square \iff 0 \times \square = 3$  is impossible b/c

1.5

$0 \times \text{anything} = 0$

so  $3 \div 0$  is undefined.

$0 \div 0 = \square \iff 0 \times \square = 0$  is undefined b/c

1.5

$\square$  can be any #.

They are undefined for different reasons, one

b/c it could be any thing, other b/c it

could not be any thing.

7. (5 points) Reza writes the following word problem for  $1\frac{3}{4} \div \frac{1}{2}$ :

"There are  $1\frac{3}{4}$  cakes left. Reza eats half of the cake that is left. How many pieces of cake did Reza eat?"

Is Reza's word problem correct for the calculation  $1\frac{3}{4} \div \frac{1}{2}$ ? If so, explain why and do the calculation to figure out the answer. If not, modify his question so that it does fit the calculation  $1\frac{3}{4} \div \frac{1}{2}$ , or make a question of your own that fits the calculation.

Reza word problem is wrong ✓ on multiple levels.

If he eats half, then it should be  $1\frac{3}{4} \times \frac{1}{2}$ , not

✓✓ divided. Also, there is no way to know how many

pieces he ate  $\frac{1}{2}$  it does not say how

the cake was sliced.

✓ Modification: Reza had some cakes, and  
✓ he ate half of the cakes. He now has

$1\frac{3}{4}$  cakes. How many cakes were there originally?



8. (5 points) A recipe calls for  $2\frac{1}{4}$  teaspoons of yeast to make one and a half pizzas. How much yeast would be needed to make 5 pizzas?

$$\begin{array}{rcl}
 \text{yeast} & 2\frac{1}{4} & \leftrightarrow 1\frac{1}{2} \text{ pizza} \\
 & \frac{9}{4} & \leftrightarrow \frac{3}{2} \\
 \times 4 \downarrow & & \downarrow \times 4 \\
 & 9 & \leftrightarrow 6 \\
 \div 6 \downarrow & & \downarrow \div 6 \\
 & \frac{3}{2} & \leftrightarrow 1 \\
 \times 5 \downarrow & & \downarrow \times 5 \\
 & \frac{15}{2} & \leftrightarrow 5 \text{ pizzas}
 \end{array}$$

$$\frac{9}{6} = \frac{3}{2}$$

15.  
 $\frac{15}{2} = 7.5$  tsp of yeast

9. (5 points) It takes 4 workers 3 hours to fill a 50 ft by 50 ft square garden with compost. Assuming all workers work at the same speed, how long will it take 6 workers to fill a 100 ft by 100 ft square garden with compost? Show/explain your work!

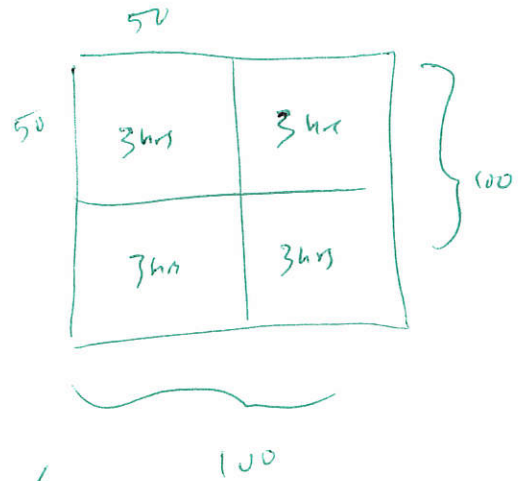
(A) 3 hours

(B) 4.5 hours

(C) 6 hours *X too small*

(D) 8 hours

(E) 12 hours *X too big*



would take 4 workers 12 hours  
to fill the 100 x 100 ft<sup>2</sup>  
garden,  
and should be less time for 6

workers  
would take 8 workers 6 hours...

$$4 \times 12 = 48 \text{ worker-hours} \quad \text{and} \quad \frac{48}{6} = \boxed{8 \text{ hrs}}$$

*3 pts for inv-prop reasoning  
2 pts for direct prop reasoning.*

10. (5 points) Let  $N$  be a number such that  $N = 6 \times p$ , where  $p$  is a prime number. How many factors does  $N$  have? Will the number of factors depend on the value of  $p$ ?

$$N = 2 \times 3 \times p$$

$p = 2$  (or  $3$ )

$$N = 2^2 \times 3$$

$$1 \times 12$$

$$2 \times 6$$

$$3 \times 4$$

6 factors

4 pts for either  $\rightarrow$   
1 pt. for "depend on  $p$ "

$p \neq 2$  or  $3$ , say  $5$ .

$$N = 2 \times 3 \times 5$$

$$1 \times 30$$

$$2 \times 15$$

$$3 \times 10$$

$$5 \times 6$$

8 factors

$N$  will have 6 or 8 factors, depending

on whether  $p = 2$  or  $3$ , or if  $p \neq 2$  or  $3$

2 pts for prime factorization but no answer

2.5 pts for " " with answer 3

11. (5 points) A solution requires 4 ml of saline for every 7 ml of medicine. How much saline would be required for 50 ml of medicine? Show/explain your work!

(A)  $28\frac{4}{7}$  ml

(B)  $28\frac{1}{4}$  ml

(C)  $28\frac{1}{7}$  ml

(D) 87.5 ml

$$\frac{50}{7} = 7\frac{1}{7} \text{ "doses" of medicine}$$

for each dose need 4 ml of saline

$$\Rightarrow 4 \cdot \left(7\frac{1}{7}\right) = 28\frac{4}{7} \text{ ml}$$

or

$$4 \times \frac{50}{7} = \frac{200}{7} =$$

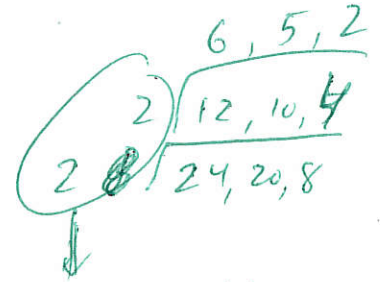
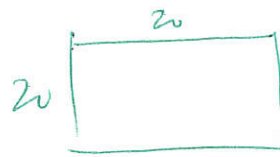
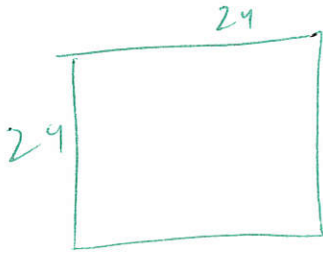
$\Rightarrow 28\frac{4}{7}$  ml

$$\begin{array}{r} 28 \\ 7 \overline{) 200} \\ \underline{14} \phantom{0} \\ 60 \\ \underline{-56} \\ 4 \end{array}$$

+ 3 for <sup>good</sup> reasoning to get to 28

2

12. (5 points) (a) Devonte has three square quilts he wants to make: one with a side length of 24 inches, one with a side length of 20 inches, and the third with a side length of 8 inches. Devonte wants to make a square pattern that he can repeat to make a design to go on all three quilts. What is the side length of the largest square pattern that will fill up all of the space in all of the squares? Explain/show your work.



Need GCF of 24, 20, and 8

GCF = 4

which is 4

7.5

- (b) Eva is filling goodie bags to give out at her birthday party. Each goodie bag will have one eraser, one pencil, and one small wombat figurine. Eva has an unlimited amount of bags, but the erasers come in packages of 8, the pencils come in packages of 24, and the wombat figurines come in packages of 20. Eva wants to buy enough items so that she will have the same number of erasers, pencils, and wombats. Assuming she cannot buy fractions of packages, what is the least amount of items she will have to get in order to have the same number of erasers, pencils and wombats?

Need LCM of 24, 20, 8

$$\begin{array}{ccc} \downarrow & \downarrow & \downarrow \\ 2^3 \cdot 3 & 2^2 \cdot 5 & 2^3 \end{array}$$

$$LCM = 2^3 \cdot 3 \cdot 5 = \span style="border: 1px solid black; padding: 2px 5px;">120$$

2 if 240

7.5

13. (5 points) Two weeks ago, both Barstucks and Baricou coffee companies sold their coffee at the same price. Then one week ago, Barstucks increased their prices by 50%, and Baricou increased their prices by 5%. Yesterday, Barstucks decreased their new prices by 40% and Baricou kept their prices the same. Who has the cheaper coffee now (or are they equal again)? Explain your reasoning and/or show your work!

Started at \$100

|              | Barstucks | Baricou |
|--------------|-----------|---------|
| one week ago | \$150     | \$105   |
| Yesterday    | \$90      | \$105   |

$$40\% \text{ of } 150 = \frac{40}{100} \cdot 150 = \$60$$

$$\Rightarrow \text{Barstucks is } 150 - 60 = 90$$

Barstucks is cheaper b/c their coffee

initially went down a total of 10%.

14. (5 points) Suppose that an even number has an odd divisor. If you divide the even number by that odd divisor, will the result be an even number or an odd number, or does it depend on what the numbers are? Explain your answer and show your reasoning. You should make sure that your answer is explained *in general*, it is NOT enough to just give some examples.

$$12 \div 3 = 4 \quad \checkmark$$

$$6 \div 3 = 2$$

$$200 \div 5 = 40$$

The result is always even.

Consider  $E \div O = \#$

$$\Leftrightarrow \text{odd} \times \# = \text{even}$$

Thus an odd ~~o~~ times our  $\#$  must be even. If our

$\#$  was odd, we would have an odd  $\#$  of

"unpaired" left-over to pair, which could not be paired up, leaving us with an odd  $\#$ .

Thus ~~o~~ we must have an even  $\#$  as our  $\#$ .

**Extra Credit**(up to 3 points) You can choose to have either 1 extra credit point or 3 extra credit points. If you choose 1 point, you are guaranteed to get 1 point. If you choose 3 extra credit points, and 3 or more students in the class (including yourself) choose 3 points, then everyone who chose 3 points will get NO extra credit.

